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Elastic Electron Scattering from the Deuteron Using the Gross Equation

J.W. Van Orden[1,2], N. Devine[2] and F. Gross[2,3]

[1]Department of Physics, Old Dominion University, Norfolk, VA 23529
[2]Continuous Electron Beam Accelerator Facility 12000 Jefferson Avenue, Newport News, VA 23606
[3]Department of Physics, College of William and Mary, Williamsburg, Virginia 23185

Abstract

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[2] *Continuous Electron Beam Accelerator Facility 12000 Jefferson Avenue, Newport News, VA 23606*
[3] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23185*
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With CEBAF now coming on line, it will be routine to probe nuclear systems with electron scattering where the energy and momentum transfers will be well in excess of the nucleon mass. Under such circumstances, the usual nonrelativistic description of the nucleus is no longer reliable, and it is necessary to develop relativistically covariant models of the nuclear system. Here, we will present a calculation of the electromagnetic form factors of the deuteron in the context of the Gross equation [1]. Our calculation is manifestly Lorentz covariant and has been constructed to be gauge invariant. We will show that by carefully constraining the interaction model to fit the nucleon-nucleon phase shifts and by using only minimal exchange currents we are capable of obtaining an excellent description of the available data.

The Gross equation [1] is a quasipotential equation [2,3] in which the relative energy is constrained by restricting one of the nucleons to its positive energy mass shell. The application of the Gross equation to the calculation of nucleon-nucleon scattering and the deuteron bound state is described in considerable detail in Ref. [4]. Calculations of these quantities were presented using a one-boson-exchange interaction kernel with several unusual features.

1. The meson-nucleon couplings include off-shell terms. For example the πNN coupling is chosen to be an adjustable admixture of pseudoscalar and pseudovector coupling.

2. The meson-nucleon-nucleon form factors depend upon the invariant masses of the three virtual particles connected to the interaction vertex. For simplicity, we assume that these form factor can be written in a factorable form [4,7]

$$F(p'^2, p^2, \ell^2) = h(p'^2)h(p^2)f(\ell^2) \tag{1}$$

where p and p' are the initial and final nucleon four-momenta, and $\ell = p - p'$ is the meson four-momentum, and $f(\ell^2)$ and $h(p^2)$ are meson and nucleon form factors, respectively. These form factors are given by eqs. (3.9) and (3.13) of Ref. [4]

3. The kernel must be antisymmetrized in order to insure that the Pauli principle is exactly satisfied.

Four models for the NN interaction were presented. Each model has been fitted to the NN phase shift data [5] and constrained so that the deuteron bound state mass is correct. The interaction model used in the calculations shown here is a variation on model IIB of Ref. [4]. The parameters of the model have been adjusted to fit the Nijmegen energy dependent np phase shifts [6], and the data base in SAID [5] gives a χ^2 per datum of 1.89 for energies of 1 to 250 MeV and of 2.53 for 1 to 350 MeV. This model uses a one-boson-exchange kernel containing six mesons: π, η, σ, σ_1, ω and ρ. The σ_1 meson is a scalar-isovector companion to the σ with a mass comparable to the σ mass. The pion mixing parameter was fixed at $\lambda_\pi = 0$ for pure pseudovector coupling. A total of thirteen parameters were adjusted in the fitting procedure.

The deuteron wave functions for this model are similar to those of model IIB shown in Fig. 2 of Ref. [4]. There are four wave functions, the usual S and D waves that appear in the nonrelativistic treatment of the deuteron and singlet and triplet P waves of relativistic origin. The contributions to the normalization of the wave function from these components are: 92.979% for the S wave, 5.015% for the D wave, 0.049% for the triplet P wave and 0.009% for the singlet P wave. The remaining 2% is associated with the derivative term in (2.71) of Ref. [4]. Note that the signs of the singlet and triplet P waves are opposite for this model.

The construction of appropriate current matrix elements for the Gross equation that maintain gauge invariance was discussed in Ref. [7]. In order to satisfy the Ward-Takahashi identities [8] in the presence of the form factors (1), an off-shell single-nucleon current operator must be introduced. A minimal form of the operator is given by [3,9]:

$$J^{(i)\mu}(p',p) = F_1(Q^2) f_0(p'^2,p^2)\gamma^\mu + \frac{F_2(Q^2)}{2m} h_0(p'^2,p^2)\, i\sigma^{\mu\nu}q_\nu + F_3(Q^2) g_0(p'^2,p^2)\,\frac{\not p' - m}{2m}\gamma^\mu\frac{\not p - m}{2m} \tag{2}$$

where

$$f_0(p'^2,p^2) \equiv \frac{h(p^2)}{h(p'^2)}\,\frac{m^2-p'^2}{p^2-p'^2} + \frac{h(p'^2)}{h(p^2)}\,\frac{m^2-p^2}{p'^2-p^2}, \tag{3}$$

$$g_0(p'^2,p^2) \equiv \left(\frac{h(p^2)}{h(p'^2)} - \frac{h(p'^2)}{h(p^2)}\right)\frac{4m^2}{p'^2-p^2} \tag{4}$$

and $F_3(Q^2)$ and $h_0(p'^2,p^2)$ are arbitrary functions subject only to the constraints that $F_3(0)=1$ and $h_0(m^2,m^2)=1$. In the calculations presented here, $h_0(p'^2,p^2) = f_0(p'^2,p^2)$ and $F_3(Q^2) = G_{Ep}(Q^2)$, for simplicity.

In constructing the current matrix element it is necessary that the on-shell constraint used in the Gross equation be consistently applied to the calculation of the current matrix element. In the case of the Gross equation, the correct expression for the current matrix element can be obtained by keeping only the positive energy nucleon poles for particle 1 in the evaluation of the energy loop integrals of the Bethe-Salpeter current matrix element. For the elastic matrix elements this leads to the Feynman diagrams displayed in Fig. 1. Here the ovals represent the deuteron vertex functions, single lines represent nucleon propagators, lines with crosses denote on-shell nucleons, and the wavey lines represent virtual photons.

Diagram 1a, where the virtual photon is absorbed on particle 2, has particle 1 constrained on shell for both the initial and final state vertex functions and can be written in the form of a matrix element of the single-nucleon current operator between two Gross wave functions. However, if the virtual photon is absorbed on particle 1, the positive energy pole can be picked up for the propagator before the absorption of the virtual photon or the one after. This leads to diagrams 1b and 1c. In these diagrams only the initial or final vertex function is on shell and

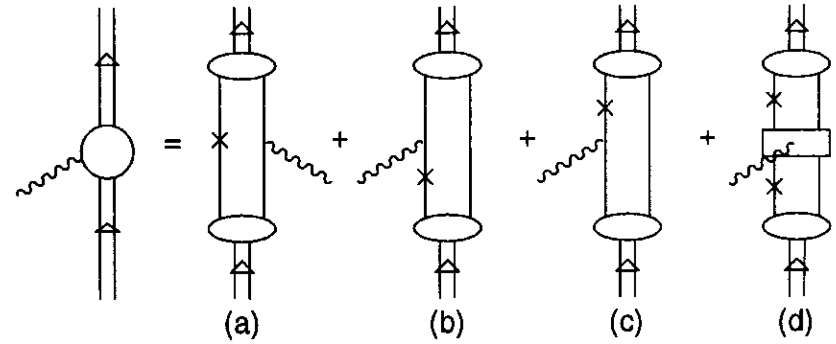

FIG. 1. Feynman diagrams representing the Gross current matrix element.

the other must be off shell. These two diagrams do not have the simple form associated with the nonrelativistic impulse approximation as does diagram 1a. The equation for the off-shell vertex function can be used to write the matrix elements for these diagrams in terms of the constrained Gross wave function [3]. The resulting diagrams may be viewed as interaction current contributions which are necessary to accommodate the on-shell constraint. It should be noted that only by calculating diagrams 1a, 1b and 1c can the proper normalization of the charge be recovered from the charge form factor in the limit $Q^2 \to 0$. Diagrams involving two-body interaction currents will have two internal energy loops which can be constrained independently to give diagram 1d, provided that the meson exchange currents are explicitly symmetrized.

Prior to Ref. [7], it was assumed that the proper form of the Gross current matrix element was described by diagram 1a along with a symmetric diagram where the photon attaches to particle 1 and particle 2 is placed on mass shell [10]. Because of the symmetry of the matrix element, the contribution of the second diagram is equivalent to diagram 1a. Thus this approximation is equivalent to simply calculating 2×diagram 1a. Since the form of this approximation looks like a matrix element of a single-nucleon current between spectator wave functions, it is referred to as the relativistic impulse approximation (RIA). Since the combination of diagrams 1a, 1b and 1c are related to the relativistic impulse approximation but represent a complete gauge-invariant description of the Gross one-body current matrix elements we will refer to it here as the complete impulse approximation (CIA).

It is possible to calculate the diagrams of Fig. 1 in two different ways. A general representation of the on- and off-shell vertex functions can be constructed in terms of an expansion in Dirac gamma matrices [11,3] and the matrix element can be evaluated as a trace in the Dirac matrix space. Alternately the various elements of the Feynman diagrams can be projected onto positive and negative energy plane wave Dirac spinors and the wave and vertex functions expanded in a partial wave basis in the deuteron rest frames. It is then necessary to boost the wave and vertex functions to the Breit frame where the calculations are performed. The calculations of the CIA have been performed using both methods and are in agreement. Due to their greater complexity,the exchange current contributions have only been calculated using the spinor expansion.

The effects of the various elements of the calculation for the impulse approximation can be seen for $B(Q^2)$ in Figure 2. The relativistic impulse approximation of Hummel and Tjon [12] with Höhler single-nucleon electromagnetic form factors [13] is shown for reference and is labelled "Tjon, RIA". Three versions of the impulse approximation are calculated using our model with the dipole parameterization of the single-nucleon form factors of Galster [14]. The calculation labelled "RIA" is as described above uses an on-shell form of the single nucleon current operator obtained by setting $f_0 = h_0 = 1$ and $g_0 = 0$ in (2). The curve labelled "RIA, off shell" is the same as the first but with the full form of the off-shell single-nucleon current operator, and the curve labelled "CIA" is the complete impulse approximation corresponding to diagrams 1a, 1b and 1c with the completely off-shell single-nucleon current operator. The use of the off-shell current operator in the RIA moves the minimum to even larger values of Q^2. The CIA is very close to the off-shell RIA (showing that the use of the RIA with off-shell currents very closely approximates the CIA) and both are in remarkably close agreement to the data. These curves show that only small contributions from the exchange currents are required to bring the CIA into good agreement with the data.

Note that for our calculation the minimum of $B(Q^2)$ at larger Q^2 than in the calculation of Hummel and Tjon. This appears to be the result of dynamical differences in the interaction models used. In particular the position of the minimum of $B(Q^2)$ is particularly sensitive to the sign of the singlet P wave v_s as can be seen by simply changing the sign of this wave function component in the calculation of the RIA as is shown in the curve labelled "RIA, $-v_s$". The effect of this change is to produce a large downward shift in the position of the minimum, without changing $A(Q^2)$ or T_{20} significantly.

The origin of the sensitivity in the minimum in $B(Q^2)$ can be seen from the expansion of the magnetic form factor to order $\frac{v^2}{c^2}$ given in Ref. [10]. The magnetic form factor is given by:

$$G_M = G_{ES} D_M^E + G_{MS} D_M^M, \tag{5}$$

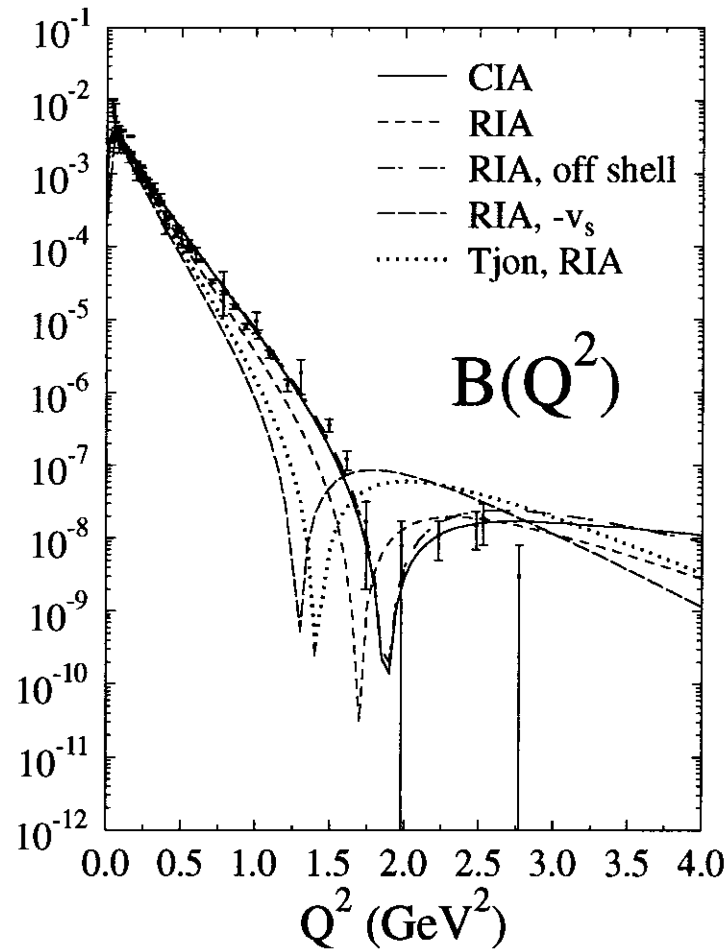

FIG. 2. $B(Q^2)$ in the impulse approximation.

where G_{ES} and G_{MS} are the isoscalar electric and magnetic single-nucleon Sachs form factors and

$$D_M^E = \int_0^\infty dr \left\{ \frac{3}{2} w^2 + \frac{2mr}{\sqrt{3}} \left[v_t \left(\frac{1}{\sqrt{2}} u - w \right) \right.\right.$$
$$\left.\left. - v_s \left(u + \frac{1}{\sqrt{2}} w \right) \right] \right\} [j_0(\tau) + j_2(\tau)] \tag{6}$$

and

$$D_M^M = \int_0^\infty dr \left[(2u^2 - w^2)\, j_0(\tau) + \left(\sqrt{2} uw + w^2 \right) j_2(\tau) \right] \tag{7}$$

with $\tau = \frac{Qr}{2}$. Here u, w, v_t and v_s are the S, D, triplet P and singlet P radial wave functions. All terms quadratic in the P-waves can be shown to be very small in this region. The position of the zero in $G_M(Q^2)$ and thus the zero in $B(Q^2)$ is sensitive to the interference terms between P-waves and the larger S- and D-waves in the second term of (6).

For elastic scattering from the deuteron, only isoscalar two-body exchange currents can contribute. The only possible isoscalar contributions for the one-boson-exchange model used here are of the type $\rho\pi\gamma$, $\omega\eta\gamma$, $\omega\sigma\gamma$, etc. These currents have couplings that are individually gauge invariant and therefore require no complicated modification of the off-shell behavior of the vertex functions and form factors in order to maintain gauge invariance. The $\rho\pi\gamma$ exchange current is related to the AAV anomaly [15] and the coupling and size of the contribution to the form factors at $Q^2 = 0$ is reasonably well constrained. The form factor for the $\rho\pi\gamma$ vertex is not known experimentally, however, and is a source of uncertainty in the calculations. The coupling constant for the $\omega\eta\gamma$ can be extracted from existing data but with less accuracy than in the previous case. The couplings and form factors for the other possible exchange currents can be predicted by quark models, but are not otherwise constrained. The contributions of the exchange currents to the elastic form factors of the deuteron have been calculated by Hummel and Tjon [12] in an approximate fashion using the Blankenbecler-Sugar equation. The form factors for all contributions were taken to be given by the vector dominance model (VMD). It was found that $\rho\pi\gamma$ and $\omega\sigma\gamma$ exchange currents were needed to obtain any agreement with the data and that contributions of the $\omega\eta\gamma$ exchange currents were small. These calculation for $A(Q^2)$, $B(Q^2)$, and $T_{20}(Q^2) = \tilde{t}_{20}(Q^2)$ are shown in Fig. 3 labelled as "Tjon, RIA+$\rho\pi\gamma + \omega\sigma\gamma$". The CIA calculation is also shown for reference. Calculations of the contributions of the $\rho\pi\gamma$ exchange current are shown for our model as calculated with the VMD form factors (labelled "CIA+$\rho\pi\gamma$ (VMD)"), and quark model form factors

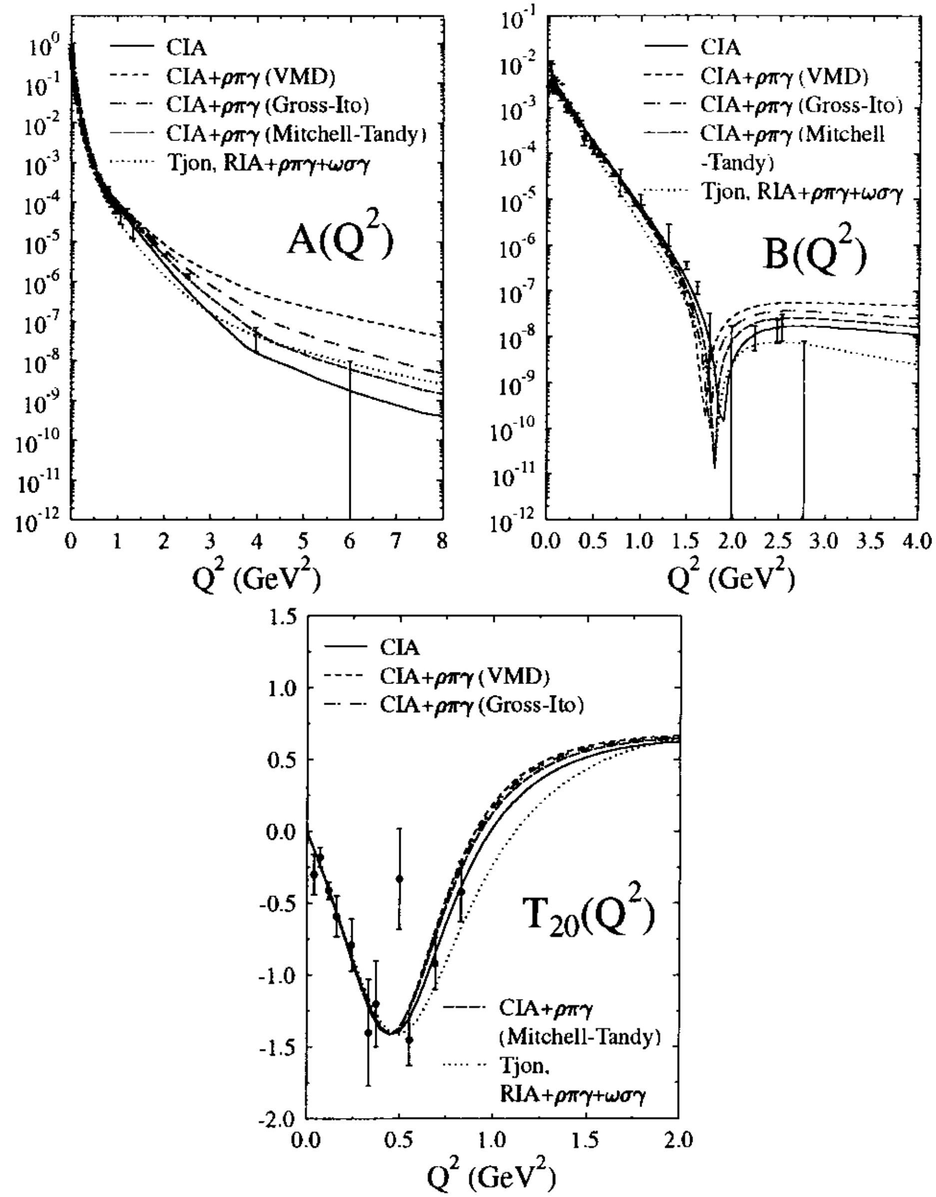

FIG. 3. $A(Q^2)$, $B(Q^2)$ and $T_{20}(Q^2)$ with exchange currents.

as calculated by Gross and Ito [16] (labelled "CIA+$\rho\pi\gamma$ (Gross-Ito)") and by Mitchell and Tandy [17] (labelled "CIA+$\rho\pi\gamma$ (Mitchell-Tandy)"). Recent calculations of the $\rho\pi\gamma$ form factor by Cardarelli, et al. [18] also give results very similar to those of Mitchell and Tandy. Both of the quark model form factors are softer than the VMD. The $\rho\pi\gamma$ exchange currents tend to increase the size of $A(Q^2)$ and to move the minimum of $B(Q^2)$ to lower Q^2. In both cases the VMD form factors produce much too large an effect while the softer quark model form factors give smaller effects. Indeed the calculation with the Mitchell-Tandy form factor is remarkably close to the data. The tensor polariztion $T_{20}(Q^2) = \tilde{t}_{20}(Q^2)$ shows some sensitivity to the exchange current contributions. The quality of the data is not yet sufficient to distinguish among the various models, however.

In summary, we have constructed a complete, relativistically covariant and gauge invariant model of elastic electron scattering from the deuteron using the Gross equation. The calculation includes the complete impulse approximation and $\rho\pi\gamma$ exchange currents. We find that the structure function $B(Q^2)$ is extremely sensitive to the presence of small P-wave components of the deuteron wave function of relativistic origin. By using a soft $\rho\pi\gamma$ electromagnetic form factor we have been able to obtain an excellent description of the data. We are presently looking for more accurate interaction models (the D/S ratio and quadrupole moment of model IIB are a little too small.) The Gross equation is also being applied to the calculation of the triton binding energy [19] and we expect that this will result in some additional constraints on acceptable models. It is possible that our best interaction models may produce different results for the deuteron form factors.

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